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GALILEO GALILEI´S THESIS EXPANDED

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Abstract

In this short article, I try to show alternative maths to real numbers in such a way that these maths (especially Transreal Numbers by James Anderson and Arithmetic of Infinity by Yaroslav Sergeyev) can also be considered as legitimate instruments for presenting the structure of reality. I call this thesis of expanding the possibilities of understanding Nature mathematically the "Galileo Galilei´s thesis extended". As an example of the application of the thesis that the mathematics that is at the base of Nature must be extended to a better assessment of the scope of physical laws, here we present the Heisenberg´s Uncertainty Principle, approached in an alternative way from a mathematical point of view.

Keywords: Heisenberg Principle; Galileu Galilei; Real numbers- Tranreal Numbers – Arithmetic of Infinity;

1. INTRODUCTION

In his 1623 book, entitled "the Assayer"[1], Galileo Galilei, unanimously considered the creator of the modern methodology of theoretical physics, states that the book of Nature is written in mathematical characters.

At the time of Galileo, such a statement had a very clear meaning: Nature is structured mathematically in points, lines and volumes; the world of Nature is a great geometric structure (Euclidean, by the way) which must be studied from this observation.

However, if we place Galileo's postulate on the structure of Nature in what was developed mathematically in History of Mathematics and which today ended up in the term "Contemporary Mathematics", how can we understand Galileo's thesis?

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[1] See Galileo Galilei, *The Assayer*, 1623. Translation from "Il Saggiatore" into English by Stillman Drake. In: http://web.stanford.edu/~jsabol/certainty/readings/Galileo-Assayer.pdf

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2. PROBLEM STATEMENT

Most likely, any current physics student, well aware of the role that mathematical language plays in the development of physics as a theory, would say that the contemporary translation of the Galilean saying is as follows: Nature is written, in its mathematical structure, with real numbers and, in some exceptional cases (as in Quantum Mechanics) an exception is made to complex numbers (if, and only if, the complex numbers can be reduced to real numbers - the concept of "observables", Hermitian operators defined in Hilbert spaces, is there to confirm this priority of the real numbers in relation to complex numbers in contexts of theoretical physics).

What may not cross the mind of a contemporary physicist student is the fact that, at the same time, there are several numerical systems or "grammars" that are alternatives to real numbers; and it should also not be known to most physics students at the most prestigious universities in the world that the thesis that Nature is written "with the real numbers" is a postulate known as Cantor's axiom, which was enunciated by Georg Cantor in 1883[2]. **Therefore,** to say that the mathematical structure of Nature coincides with the structure of real numbers, in its most varied forms of presentation, is not an empirical result, but rather the postulated "condition of possibility" of the mathematical expression of Nature itself; it is, therefore, a prior thesis on Nature, not an empirical one.

We can then ask ourselves whether Galileo's thesis understood today, that Nature is written in mathematical characters, leads us necessarily to the thesis that the mathematics underlying Nature's structure is based exclusively on real numbers and complex numbers, on what these ones have in common with the real numbers. For me, clearly Galileo Galilei's thesis that Nature is written in mathematical characters is not equivalent to the statement that the mathematics that underlies Nature is exclusively based on real numbers and their auxiliaries (complex numbers). I think that the mathematical character of Nature is not reduced to what is measurable or metrizable, to what is metaphorically associated with the use of rulers or compasses (if so, the real numbers - the "allegory" par excellence of the notion of measure or variation (deltas), would be enough); but this is not the case: Nature has a mathematical dimension that is of a metaphysical character[3], and this is revealed in statements of theoretical physics in which infinites or indeterminations appear; such statements are "intractable" by real numbers and, for this reason, are considerable meaningless or indicative of some physical limit of Nature.

3. RESEARCH QUESTIONS

But the limits are on the grammar of real numbers, not on Nature itself. For example, when we analyze Heisenberg's Uncertainty principle, we can conclude that it is impossible for an observer to accurately measure the position of a particle at any given moment. This impossibility is based on the fact that, if we consider the existence of an observer with such "epistemic power", then this same observer would verify in his/her measurements that the linear momentum of that particle would be completely indeterminate. Mathematically, according to Heisenberg's Uncertainty Principle, the conjunction of absolute precision of the position of a particle with the total indeterminacy of its linear momentum gives rise to the mathematical expression $0.\infty$, which does not refer to any real number, since it is an indeterminacy in the real numbers: there is no real number "accurate and unique" that is equal to $0.\infty$. But the Uncertainty Principle states that the joint consideration of the indeterminations of the measurements of the position of particle and its linear momentum must be a number that is not less than the real number

$$h/4\pi,$$

in which the symbol h represents Planck´s constant whose value is

$$6.62607004 \times 10^{-34} \text{ m}^2 \text{ kg / s}.$$

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[2] See EHRLICH, P. [1994]

[3] The "metaphysical character" of Nature is seen here as equivalent to the thesis that we cannot explain exhaustively Nature without appealing to infinity and to the Indeterminate, concepts that has no correspondent in the usual way we consider what is a measurement.

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So, due to an impossibility inherent to the grammar of real numbers, since there is no real number equal to $0.\infty$, and therefore $0.\infty$ is not less than $h/4\pi$, then we infer that the interdiction given in real numbers is a physical interdiction: the limitation of the grammar of real numbers becomes a limitation of the physical world.

However, there are numerical systems in which $0.\infty$ it is not indeterminate. For example, in transreal numbers, the multiplication $0.\infty$ is equal to the number *Nullity*, which in turn is not less than $h/4\pi$.

1. Transreal numbers, symbolized by $\mathbb{R}^{\boldsymbol{T}}$, consist of an extension of the real numbers[4]. Beginning with the real numbers, which can be seen as a line segment that grows indefinitely both to the right (positive real numbers whose magnitude can be as large as we like) or to the left (negative real numbers whose magnitude can be as large as we want), in which we can conceive 0 as the origin of the real numbers, transreal numbers arise by the introduction of three new numbers, namely:

a) $\frac{1}{0} = \infty$ (positive Infinity);

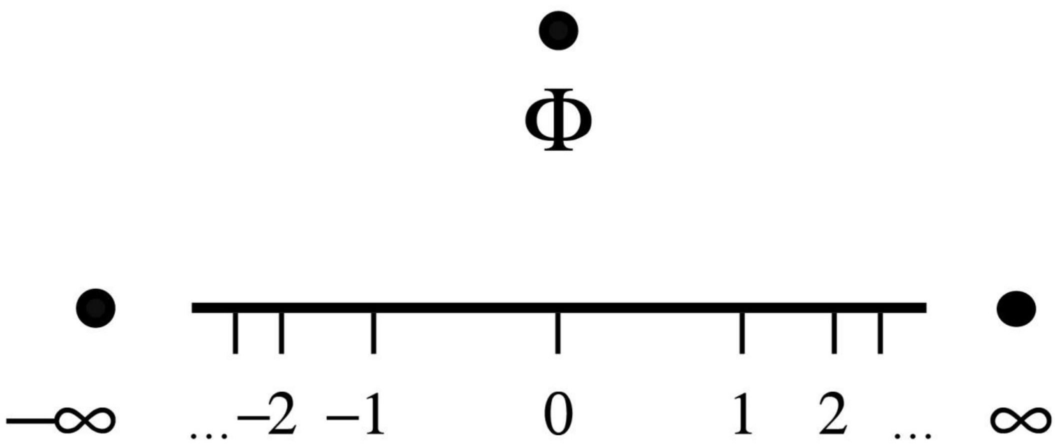
b) $-\frac{1}{0} = -\infty$ (negative Infinity);

c) $\frac{0}{0} = \Phi$ (Nullity).

Thus, transreal numbers are defined as a union between real numbers and the set composed of these three new numbers:

$$\mathbb{R}^{T} = \mathbb{R} \cup \{\infty, -\infty, \Phi\}$$

and can be viewed as follows:



In fact, Nullity, simbolyzed as **Φ,** is a number that does not maintain any order relation with any other transreal number: for any transreal number *x*, the following condition holds:

[4] On Tranreal Numbers, see ANDERSON, J, GOMIDE, W & DOS REIS, T. [2015].

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$$x \nless \Phi \text{ and } x \ngtr \Phi$$

In transreal arithmetic, it is easily demonstrated that

$$\Phi = 0/0 = 0.\infty$$

Thus, according to the condition expressed above:

$$\Phi = 0.\infty \nless h/4\pi$$ [5]

Thus, if Nature were written with the grammar of transreal numbers, it would be plausible to affirm the possible existence of an observer who measured the position of a particle with extreme accuracy to the detriment of the complete indeterminacy of the linear momentum.

Even if we postulated the existence of an observer that measured the position of a particle with infinitesimal precision, say ε, things would not be better, since the multiplication of an infinitesimal quantity ε (given epistemically as actually existing, and not as a *metaphor* of limit process that tends to zero) by an absurdly large real number is not defined in the real numbers. Thus, we would continue to affirm that there is no epistemically omniscient observer based on the limits of the grammar of real numbers.

However, there are numerical systems, such as the Arithmetic of the Infinity created by Yaroslav Sergeyev[6], in which the multiplication of an infinitesimal quantity by a very large number or by an infinite quantity is defined.

Basically, the Arithmetic of Infinity postulates that there is a greater natural number, called *grossone* and represented as ①, in such a way that the set of natural numbers ℕ, in its entirety, can be presented as follows:

$$\mathbb{N} = \{1,2,3, \ldots, \frac{①}{2} - 1, \frac{①}{2}, \frac{①}{2} + 1, \ldots, ① - 2, ① - 1, ①\},$$

So, in the Arithmetic of Infinity, the natural numbers are divided into two disjoint sets: the finite numbers n and the infinite numbers of the form $j\,(\frac{①}{k}) \pm \boldsymbol{m}$, such that $j, k \in \mathbb{N}$, and $j/k < 1$; $m \in \mathbb{N} \cup \{0\}$. Every infinite number of the form $j\,(\frac{①}{k}) \pm \boldsymbol{m}$ is less than ①.

[5] In one of its simplest ways to express it, the Heisenberg uncertainty principle states that the joint indeterminacies of the measures of two conjugate physical quantities are always greater than or equal to $h/4\pi$.
To be considered a true statement even in the borderline cases where we operate with joint indeterminacies equal to zero and infinity, within transreal arithmetic, the principle must be modified to its equivalent form which states that the joint indeterminacies are not less than $h/4\pi$.

[6] On Arithmetic of the Infinity, see Sergeyev, Y. [2017]

In Sergeyev´s system, the following identities link the *grossone* ① to the elements 0 and 1(see SERGEYEV, *Op. Cit*, p. 236):

1- 1. ① = ①. 0 = 0
2- ① - ① = 0.
3- $\frac{①}{①} = 1.$
4- $①^0 = 1.$
5- $1^{①} = 1.$
6- $0^{①} = 0.$

Among the identities presented above, worthy of note is **3**: it states that the division of an infinite number, in this case ①, by an infinitesimal number, 1/①, is equals to 1.

Thus, by interpreting the uncertainty principle of Heisenberg from the arithmetical identities that are true in the Arithmetic of Infinity, *if we postulate that the particle position measurement is done with infinitesimal precision and the indeterminacy of the linear momentum is infinite and of grossone size* - the infinite unit present in the Arithmetic of Infinity that counts the totality of natural numbers -, then the multiplication of these quantities is equal to 1, which is obviously not less than $h/4\pi$.

Thus, if the mathematics of Nature were based on the Arithmetic of the Infinity and the treatment that this arithmetic gives to measurements is accepted, then would be plausible to have an observer who, according to Heisenberg's Uncertainty Principle, would be able to measure the position of a particle with complete precision, while verifying the total indeterminacy of the linear momentum of such particle.

4. CONCLUSION.

Thus, as a conclusion to this short article, I launch the following "metaphysical" hypothesis about Nature (an extension of Galileo's thesis):

The book of Nature is written in a Mathematics whose grammar does not dispense or render without physical significance quantities with infinite or indeterminate values; these infinite or indeterminate values are indicative that Nature, in its mathematical structure, is not reduced to what is actually measurable, but has something more than that - something "metaphysical", so to speak.

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